

Asset Management Area

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Risk Management Report

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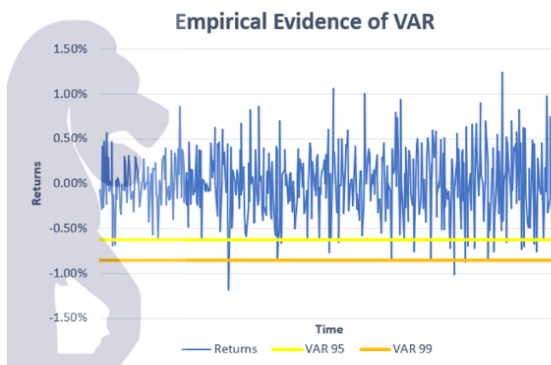
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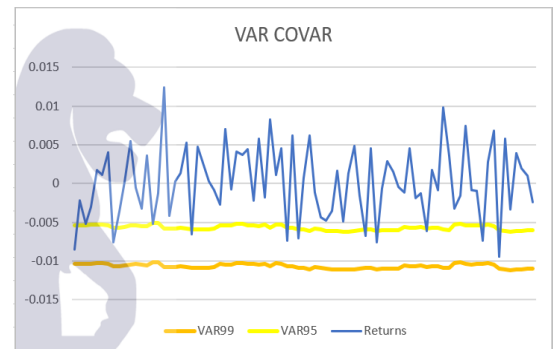
I. Main Results

Methods	Historical	Variance Covariance	Monetacarlo Simulations	Bootstrapping
VaR ($\alpha = 5\%$)	-630 €	-570 €	-710 €	-640 €
VaR ($\alpha = 1\%$)	-860 €	-1.068 €	-1.020 €	-760 €

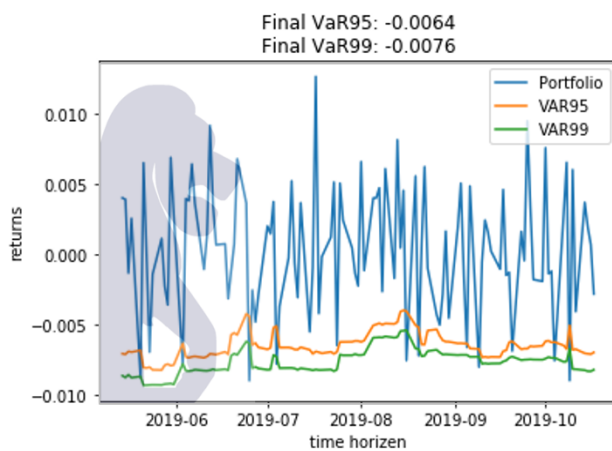
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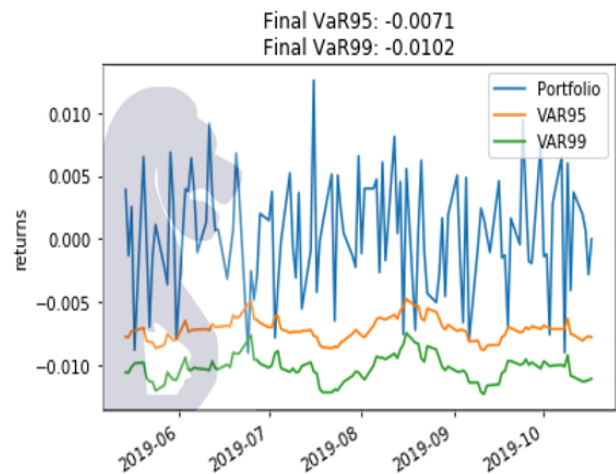
Variance-Covariance with rolling windows



Montecarlo Simulation with rolling windows



Bootstrap with rolling windows



Warning

II. Introduction

The main objective of this report is to quantify the risk embedded in the Minerva IMS Active Portfolio built by the Portfolio Management Team. We use a daily perspective on the potential extreme behaviour of a basket of assets, with the future goal of sending signals to portfolio managers to marginally review their selection of stocks or the weights assigned to each asset. As an Investment Risk division, our focus is the estimation of two main risk indicators:

- The daily Value at Risk (VaR): the maximum portfolio loss that occurs with $\alpha\%$ of probability over a time horizon of 1 day. For instance, if the VaR ($\alpha=5\%$) = -3.00%, it means that tomorrow there is a 5% probability of encountering a loss in the interval [-100%, -3.00%] potentially.
- The daily Expected Shortfall (CVaR): the expected return on the portfolio in the worst $\alpha\%$ of cases. So, it is just a weighted average of the returns lower than the VaR.

We downloaded the historical daily prices (from 20/10/2017 to 17/10/2019) of the 78 stocks that compose Minerva IMS Active Portfolio, and for each, we computed daily log returns over time. Hence, our panel dataset is composed of 520 observations (*i.e.* log returns) for every stock. Minerva IMS Active Portfolio is a so-called “Zero-Investment Portfolio” built through factor investing with 39 long positions and 39 short positions, whose inception date is 12/04/2019. Please read Minerva Asset Management Report for more details about the composition of the Active Portfolio.

In the historical approach we used the data to estimate the risk encountered by the portfolio in the time range selected. The VaR and the Expected Shortfall that come from this analysis are two measures that are relevant only for the dataset considered. In fact, to use them as a relevant indicator for the future trends of our portfolio we need to assume that the past can explain the future, an assumption that is always difficult to demonstrate.

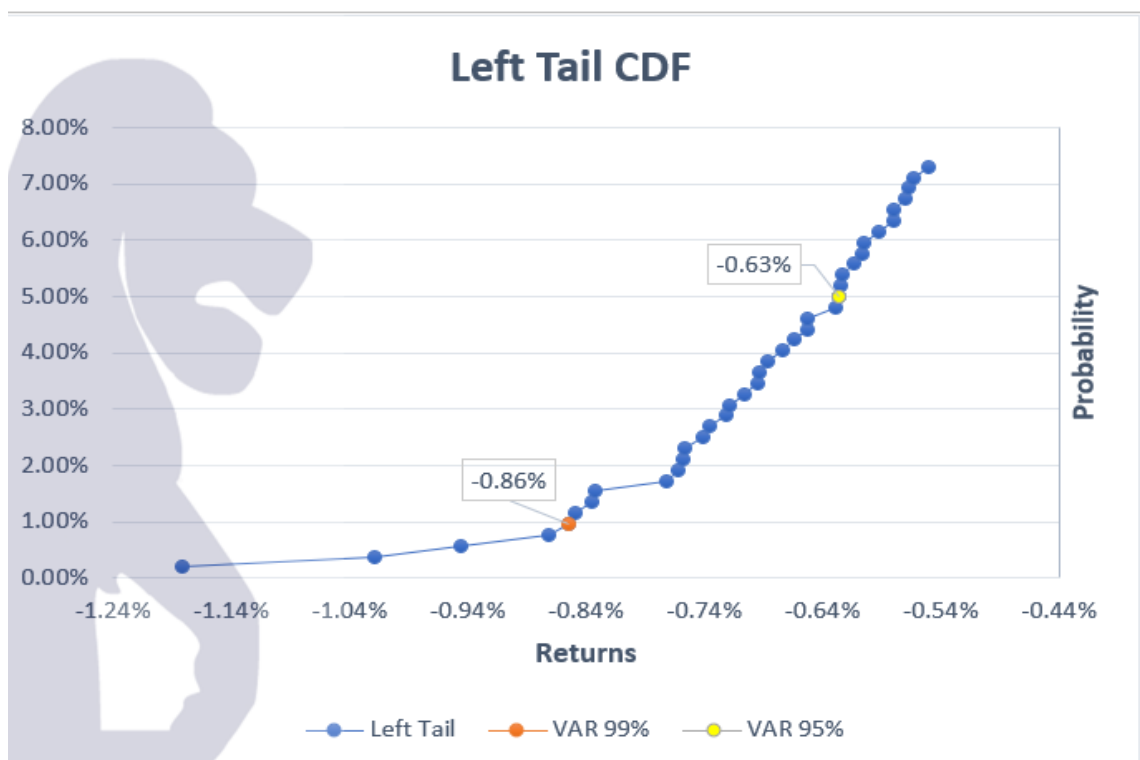
For this reason, we performed an analysis based on the theoretical distribution of the returns of the stock in the portfolio. We assume for a first approach that the log-returns of the stocks have a multivariate normal distribution, and then we performed the Var-Cov analysis on 135 data points (it will better represent the returns of the portfolio). The analysis consists of Value at Risk analyses that will be performed on homogeneous rolling windows of 52 consecutive log returns for every stock. A generic rolling window is a subset of the full dataset that contains 52 consecutive observations for every stock. Since the full dataset has 134 log returns for every stock and a single rolling window contains 52 consecutive observations, VaR computation will be performed on 82 rolling windows. By construction, rolling windows can be ordered in time from the most recent to the least recent. Computing daily Value at Risk for each rolling window will provide us with 82 different estimates.

III. Historical Analysis

This approach is valuable since it does not require any assumptions on the distribution of returns. We simply suppose that returns are identically and independently distributed. This estimate is robust to different hypothesis but, on the other side, is suffers from high sampling variability.

- Value at Risk (VAR)

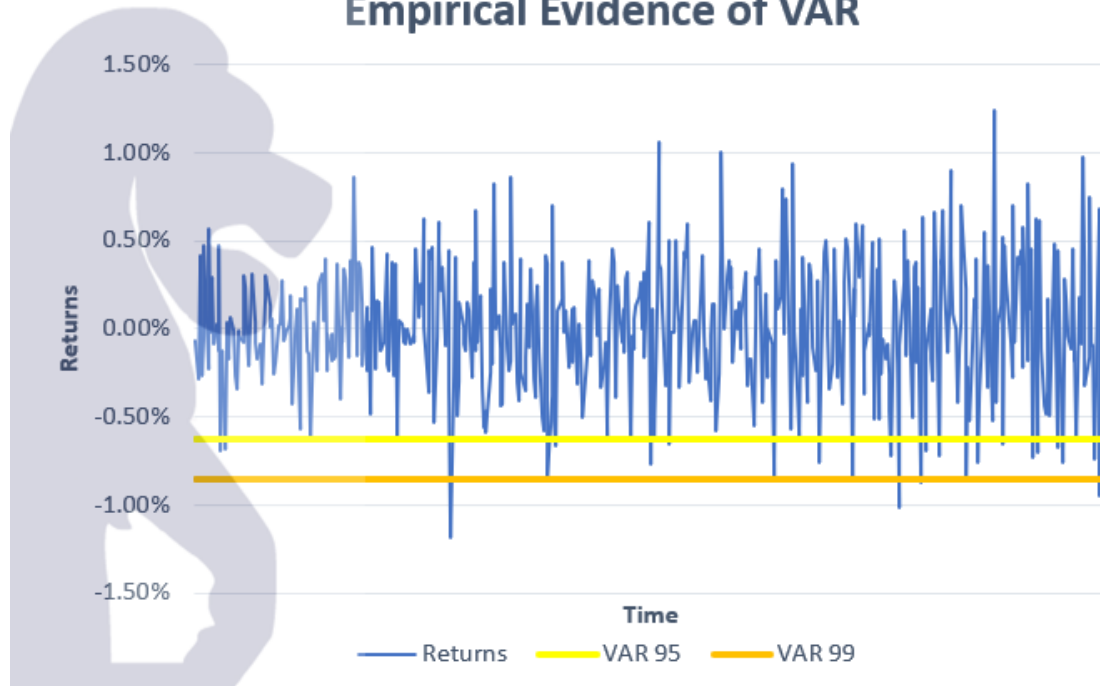
For computing the estimates of the VaR and the Expected Shortfall we used just a few lines of VBA code. Here it is shown the left tail of the distribution given by the historical data with the highlight of the $\text{VaR}(\alpha=5\%)$, called VAR 95, and of the $\text{VaR}(\alpha=1\%)$, called VAR 99.



This means that, if we decide to invest 100,000.00 € in the portfolio, there is a daily probability of 5% of losing 630 € or more, and a 1% probability of losing 860€ or more; these are very small amounts compared to what we have invested, so the downside risk seems to be very low.

Moreover, those estimates are confirmed by looking at the entire series of return in the period considered in the graph below.

Empirical Evidence of VAR



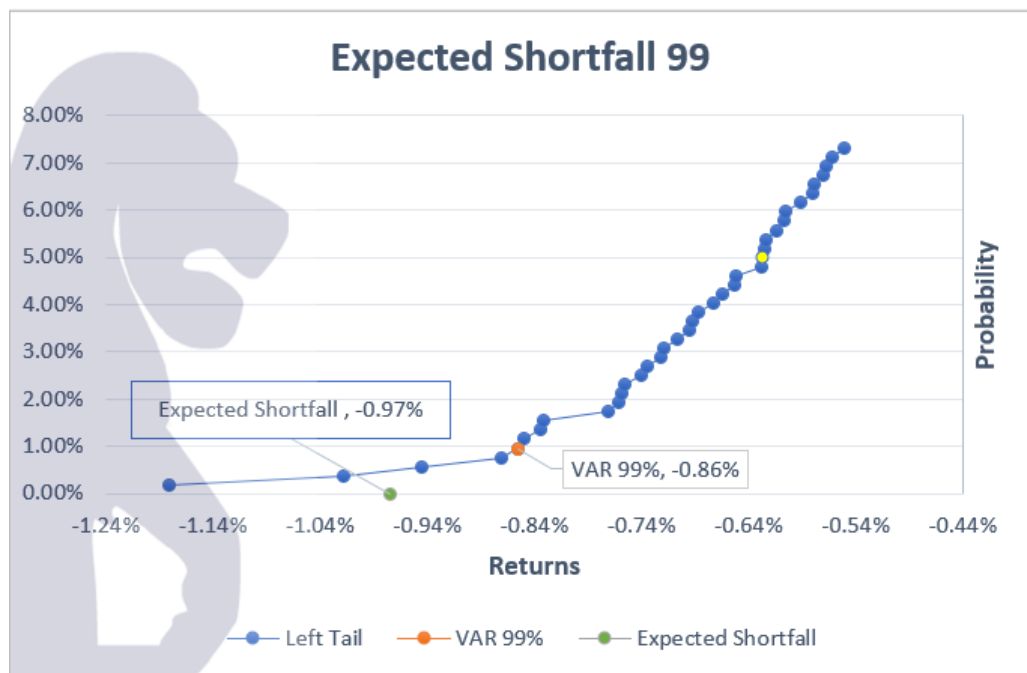
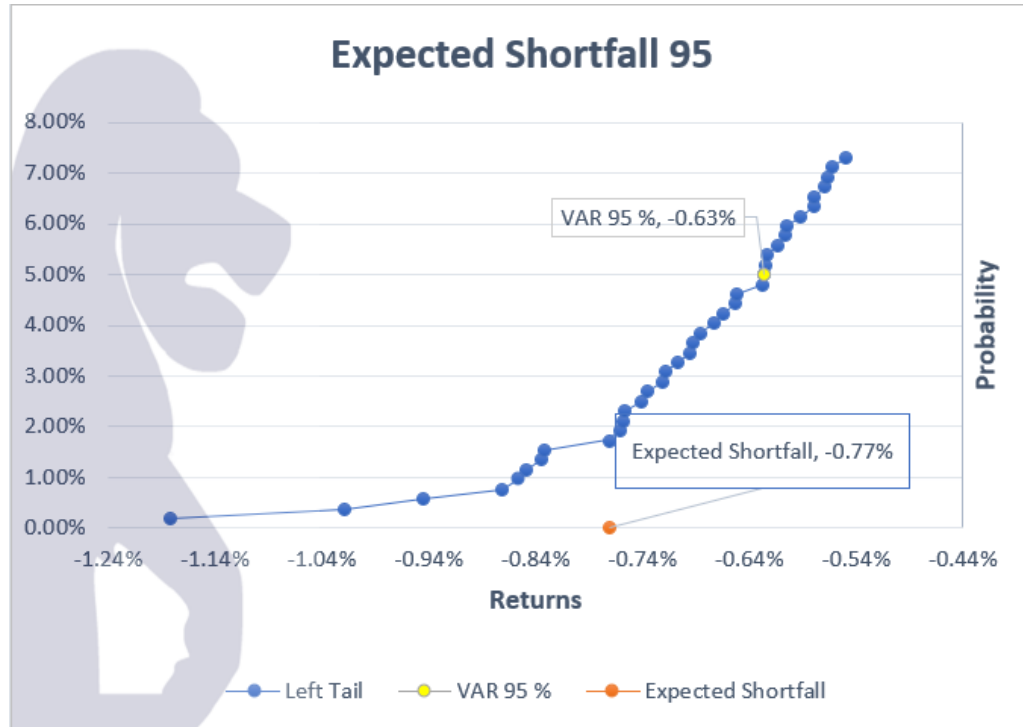
As it is evident, the line of the VAR 99 is crossed only a few times (in 1% of the occurrences) and the same it happens for the VAR 95.

- Expected Shortfall (CVAR)

As we have stated before, the Expected Shortfall is an average of the values lower than the VaR and for this reason it is called also Conditional VaR (CVAR). This risk measure is relevant when the left tail of our distribution is not so regular and to gather information from the lowest observation. It should be read after estimating the corresponding VaR in the following way:

- If we observe an Expected Shortfall near the Var, it means that the observations below the VaR are close to it, and so we are restricting the range of possible losses
- If the Expected Shortfall is far from our VaR, it means that there are relevant possibilities of encountering losses greater of our first estimate and it represents a warning

Here there are the graphs relative to the 5% and 1% cases.



In monetary terms, with a 100,000.00 € investment, in 5% of the cases we have an expected loss of 770€, while in the worst 1% we could expect a loss of 970€.

<i>Summary</i>	<i>VAR</i> <i>99%</i>	<i>VAR</i> <i>95%</i>	<i>CVAR</i> <i>99%</i>	<i>CVAR</i> <i>95%</i>
Percentage loss	-0.86%	-0.63%	-0.97%	-0.77%
Monetary loss	860 €	630 €	970 €	770 €

IV. Normality Hypothesis

- Rolling VAR-COV

Under the assumption that log returns of each stock are normally distributed, we implemented rolling variance-covariance approach to estimate Portfolio VaR.

For a generic rolling window of 52 observations, the procedure can be summarized as follow:

- 1) We estimated historical standard deviations and correlation matrix of the stocks in our portfolio.
- 2) According to the *RiskMetrics* procedure, we set the expected return of every asset to be equal to 0. Hence, exploiting Normality assumption, we computed the alpha quantile r_α of the return distribution of every stocks as

$$r_{i,\alpha} = \sigma_i \times z_\alpha \quad \forall i = 1 \dots 78$$

where σ_i is the estimated standard deviation of the i^{th} stock and z_α is the alpha quantile of a standard normal distribution $Z \sim N(0; 1)$. Note that using rolling windows technique, we did not constrain the standard deviations of each stock to be constant throughout the analysis rather we allow it to change. In fact, there is no reason to believe that the standard deviation of a random stock in rolling window 0 is the same as the standard deviation of the same stock in rolling window 100 because of the three months gap between the rolling windows.

- 3) We obtained an estimate of Value at Risk for each single position in our portfolio multiplying the alpha quantile of a stock by the monetary amount invested in it. Since our portfolio include an equal number of long and short positions, and the amount invested in each stock is constant and equal to [2564.10€], the alpha quantile of a long position is multiplied by 2564.10€ while the alpha quantile of a short position is multiplied by -2564.10€. Assuming to have a long position in the first 39 stocks in our portfolio and a short position in the last 39:

$$\begin{aligned} VaR(i) &= r_{i,\alpha} \times (2564.10) \quad \forall i = 1 \dots 39 \\ VaR(j) &= r_{j,\alpha} \times (-2564.10) \quad \forall j = 40 \dots 78 \end{aligned}$$

where $VaR(i)$ is the Value at Risk of a generic long position and $VaR(j)$ is the Value at Risk of a generic short position.

- 4) We aggregated each single position VaR into a row vector $\mathbf{w} = [VaR(1), VaR(2), \dots, VaR(78)]$ and, according to the Variance-Covariance approach, we computed Portfolio Value at Risk as:

$$Portfolio VaR = \sqrt{\mathbf{w}} \times CORR \times \sqrt{\mathbf{w}'}$$

where $CORR$ is the estimated correlation matrix of the 78 assets in the portfolio.

- 5) Every step of the above procedure has been iterated through recursive techniques (such as loops) for every rolling window of our analysis, and for $\alpha = 5\%$ and $\alpha = 1\%$.

Therefore, the outputs of this procedure are two column vectors which contain Portfolio VaR for every rolling window considered:

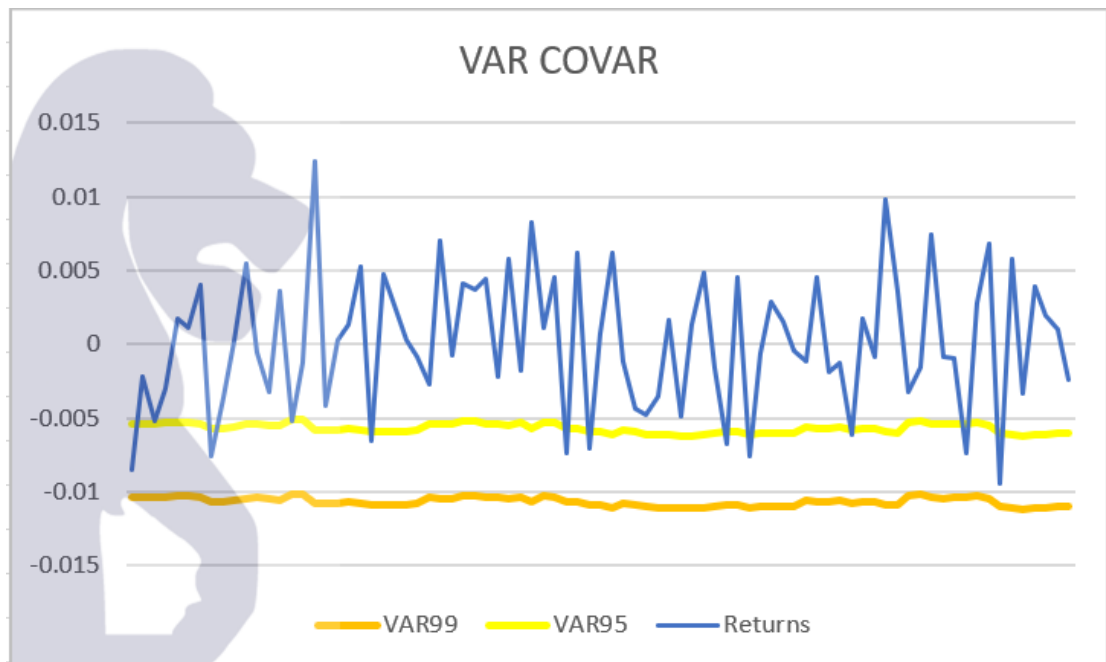
$$\mathbf{PortfolioVaR}(\alpha = 5\%) = [\mathbf{PortfolioVar}_0(5\%), \dots, \mathbf{PortfolioVar}_{-82}(5\%)]'$$

$$\mathbf{PortfolioVaR}(\alpha = 1\%) = [\mathbf{PortfolioVar}_0(1\%), \dots, \mathbf{PortfolioVar}_{-82}(1\%)]'$$

where the first element of each vector is the most recent estimate of Portfolio Value at Risk, having been computed using the observations of the first rolling window (rolling window 0), and the last element of this vector is the oldest estimate of Portfolio Value at Risk, having been computed using the observations of the last rolling window (rolling window -82).

Plotting the vectors of VaR (expressed in terms of returns) will both give a graphical idea of the time evolution of this risk estimates. Moreover, the graph below is an attempt to carry on an intuitive backtesting procedure: whenever the blue line crosses the yellow line, the portfolio under analysis experienced a loss greater or equal than the estimated monetary VaR ($\alpha = 5\%$). Similarly, whenever the blue line crosses the orange line, the portfolio under analysis experienced a loss greater or equal than the estimated monetary VaR ($\alpha = 1\%$).

Carrying on this visual procedure, we conclude that Variance-Covariance VaR approach led to a reliable risk estimates when the confidence level was settled at 1%.

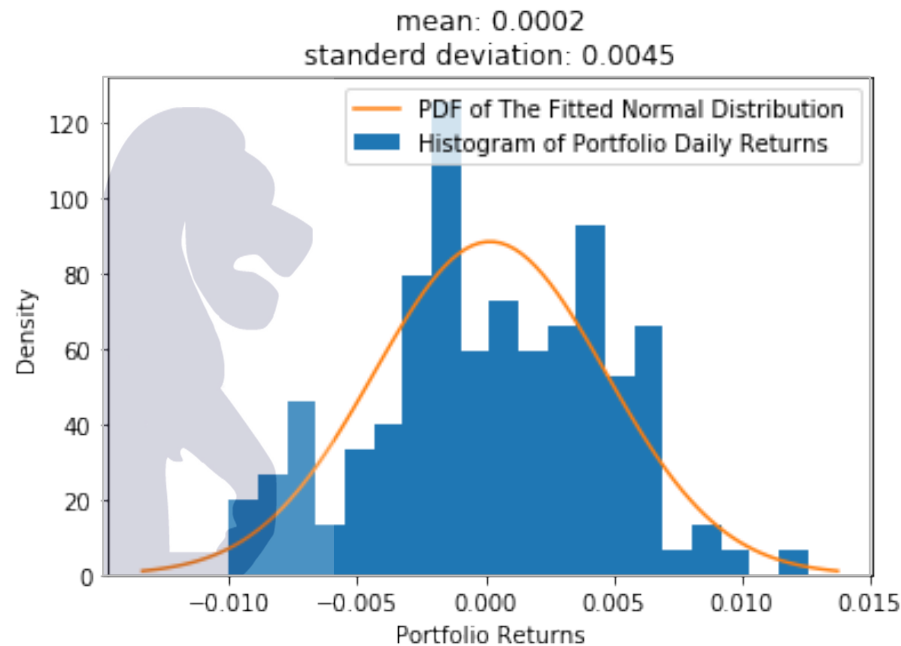


For every rolling window, *i.e.* for every set of 52 consecutive observations, we have an estimate of Portfolio VaR. However, it is necessary find the number that better approximate the overall daily risk exposure of our portfolio in the next month. One possibility would be picking the first element of the vector **PortfolioVaR**($\alpha\%$) , which is the VaR estimated using the most recent rolling window. However, since the above graph shows that VaR has been volatile throughout the rolling windows considered, we believe that an exponentially smoothed average of the elements of the vector **PortfolioVaR**($\alpha\%$) is the best approximation of the overall daily risk exposure of our portfolio in the next month. With a discount factor $\lambda = 0.94$, the final estimates of Portfolio VaR, for $\alpha = 5\%$ and $\alpha = 1\%$, are:

$$Final VaR(\alpha = 5\%) = \frac{\sum_{i=-82}^0 PortfolioVar_i(5\%)}{82} = 570\text{€}$$

$$Final VaR(\alpha = 1\%) = \frac{\sum_{i=-82}^0 PortfolioVar_i(1\%)}{82} = 1068\text{€}$$

- Normality Test



The histogram above shows that Portfolio Returns share some typical trends of high frequency returns that differentiate their empirical density function from a Gaussian distribution. Indeed, the empirical distribution of Portfolio Returns, approximated by the histogram, is more peaked around zero than a Gaussian distribution with the same mean and variance. Moreover, a positive excess kurtosis is a quantitative signal of the so called “fat tails” phenomenon: extreme returns have occurred with a relatively high frequency. If the Portfolio Returns distribution were gaussian, those extreme events would have occurred with a lower frequency. Lastly, the Portfolio Returns distribution is slightly positively skewed.

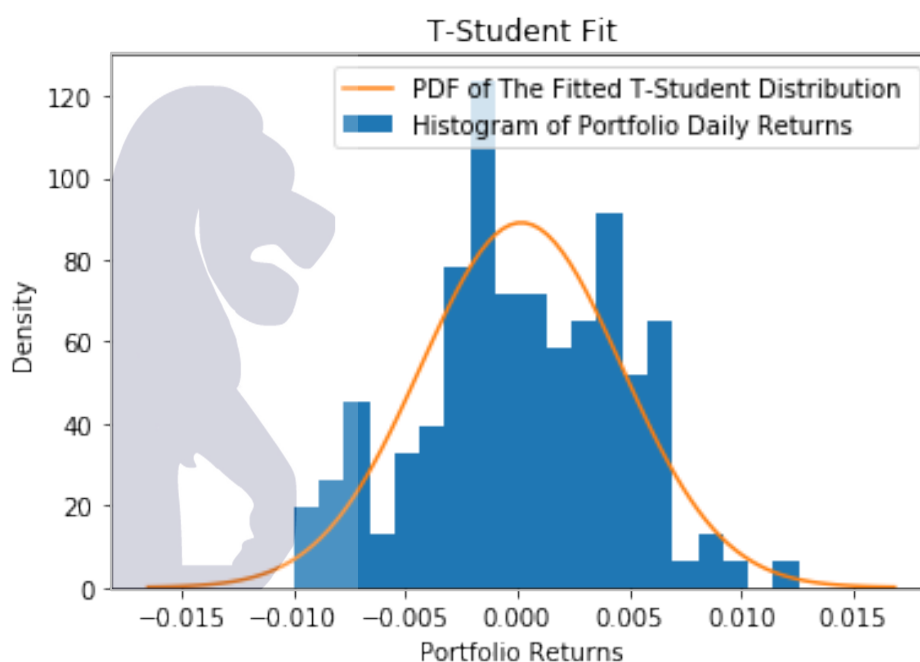
It is necessary to perform normality tests on the distributions of log returns of the stocks in our portfolio because Variance-Covariance approach requires multivariate normality of log returns distribution. This means that the joint probability density function of the log returns of the 78 stocks in our portfolio must follow a multivariate normal distribution. A set of random variables (log returns in our case) follows a multivariate normal distribution if and only if every random variable is marginally gaussian and any linear combination of them is normally distributed. as well.

After having performed the Shapiro-Wilk normality test we cannot reject the normality hypothesis and we conduct our further analysis with a multivariate normal distribution. The output of these tests is omitted for the sake of brevity.

V. Montecarlo Simulations

- Rolling Montecarlo

Because of the above-mentioned normality tests, we keep the Gaussian hypothesis. For the sake of completeness, we also fit a T-student distribution to see if it could be a better representation of the returns.



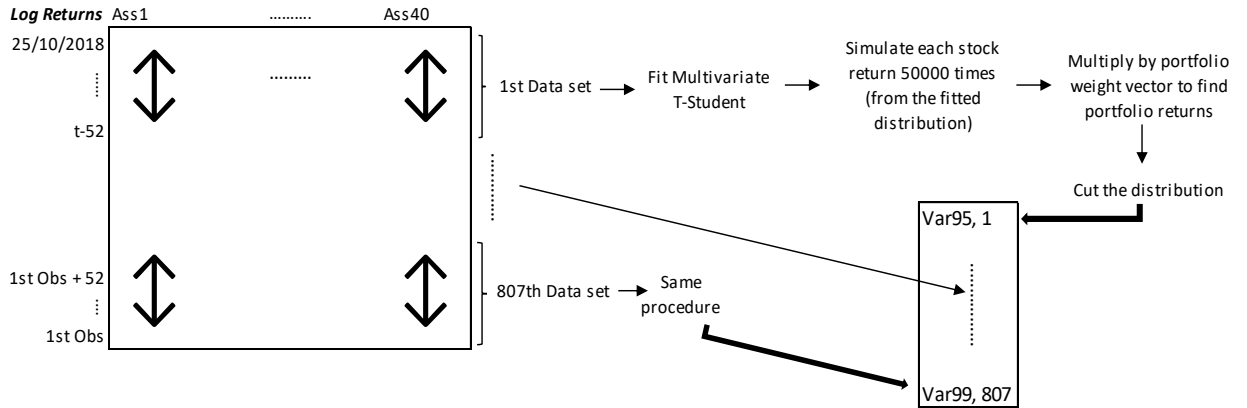
To this end, we now move to the simulation approach:

We are asking Python to:

- fit rolling data with 115 Multivariate Normal distributions, containing 78 stocks and 21 data points each
- simulate values for each stock and for each roll (for each Multivariate distribution) 50000 times
- assemble simulating values into portfolio values using portfolio weights and/or € amounts
- cut the distribution at the desired percentile

The procedure to obtain these results is much more straight-forward from a coding perspective compared to the Var-Cov approach, since we need one single loop.

Here is how loop works:

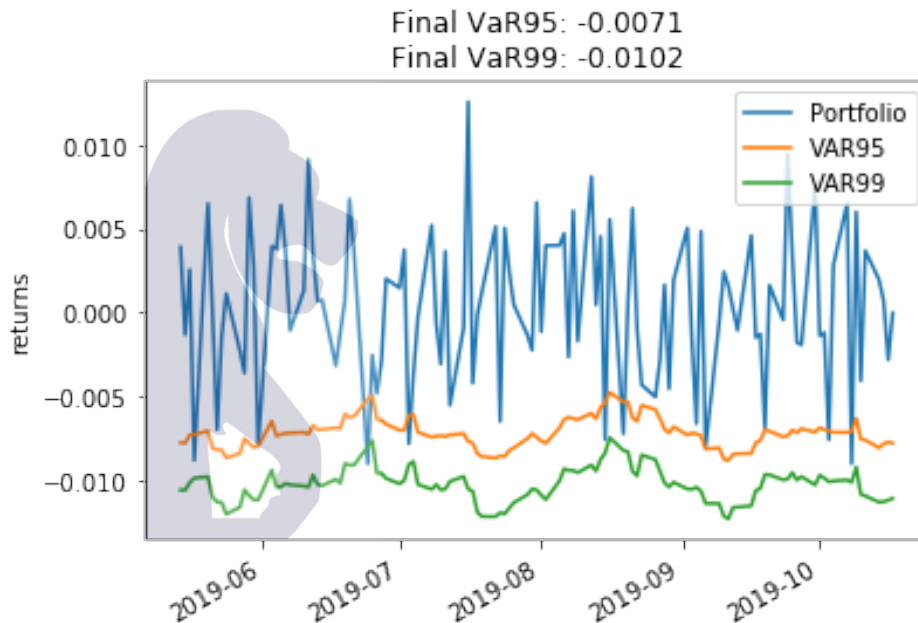


Therefore, the outputs of this loop are two column vectors which contain Portfolio VaR for every rolling window considered:

$$\begin{aligned} \text{PortfolioVaR}(\alpha = 5\%) &= [\text{PortfolioVar}_{-115}(5\%), \dots, \text{PortfolioVar}_0(5\%)]' \\ \text{PortfolioVaR}(\alpha = 1\%) &= [\text{PortfolioVar}_{-115}(1\%), \dots, \text{PortfolioVar}_0(1\%)]' \end{aligned}$$

where the last element of each vector is the most recent estimate of Portfolio Value at Risk, having been computed using the observations of the first rolling window (rolling window 0), and the first element of this vector is the oldest estimate of the Portfolio Value at Risk, having been computed using the observations of the last rolling window (rolling window -115).

Plotting the vectors of VaR (expressed in terms of returns) will both give a graphical idea of the time evolution of this risk estimates. Moreover, the graph below is an attempt to carry on an intuitive back testing procedure: whenever the blue line crosses the orange line, the portfolio under analysis experienced a loss greater or equal than the estimated monetary VaR ($\alpha = 5\%$). Similarly, whenever the blue line crosses the green line, the portfolio experienced a loss greater or equal than the estimated monetary VaR ($\alpha = 1\%$). Carrying on this visual procedure, we conclude that Montecarlo VaR approach led to a reliable risk estimates when the confidence level was settled at 1%.



We then compute the mean of the VaR vector to obtain the Final VaR return for $\alpha = 5\%$ and $\alpha = 1\%$, we then multiplying by 100,000 to get the euro loss amount:

$$Final VaR(\alpha = 5\%) = \frac{\sum_{i=-82}^0 PortfolioVar_i(5\%)}{82} = 710\text{€}$$

$$Final VaR(\alpha = 1\%) = \frac{\sum_{i=-82}^0 PortfolioVar_i(1\%)}{82} = 1020\text{€}$$

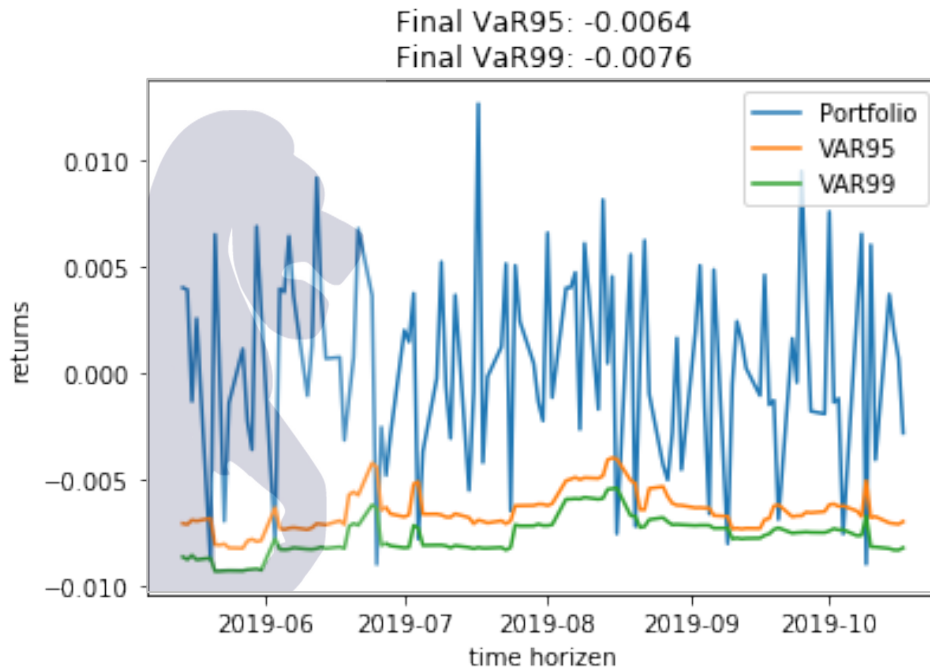
- Bootstrapping

To have a better idea on what a Monthly VaR would look like for this active portfolio, we implement a Bootstrapping procedure. Bootstrapping stands in the middle between Montecarlo Simulations and Historical Simulation, it combines a random element (the return is drawn randomly) and a historical element (the return is drawn from a historical time series).

How it works:

It is a sampling procedure with replacement from the original data. In order to implement it we coded two loops able to provide a rolling VaR measure like the other methodologies. For each estimate we use the data of the prior month, we extract 21 random data points and compute the non-parametric VaR 1000 times. After that, we take the mean and use this as a measure of the real VaR for that time. This is done for all 115 (136 - 21) of the possible timestamps. In this way, we are overcoming the main limitation of historical VaR: in fact we are simulating a time series that is consistent with the original one but at the same time we do not have constraints in terms of number of observations (length of the sample).

By plotting our estimated VaR in respect to the returns, we can get a general idea of how well the estimation replicates probable losses:



In order to get the final VaR for both quantiles we simply compute the average over the 115 timestamps and multiply by 100,000 which is our portfolio value:

$$Final VaR(\alpha = 5\%) = \frac{\sum_{i=-115}^0 PortfolioVar_i(5\%)}{115} = 640\text{€}$$

$$Final VaR(\alpha = 1\%) = \frac{\sum_{i=-115}^0 PortfolioVar_i(1\%)}{115} = 760\text{€}$$



Warning

This is an academic paper related to an academic project. This paper is not an investment suggestion and it does not in any way represent an invitation to purchase the securities we virtually invest in.